Reading 6: The Time Value of Money

LOS 6a: interpret interest rates as required rates of return, discount rates, or opportunity costs

Time value of money is a concept that refers to the greater benefit of receiving a given amount of money at present rather than in the future, due to its earning potential. Money could be invested in a bank account and earn interest even for an overnight period. Interest earned will depend on the rate of return offered by government bonds (risk-free assets), inflation, liquidity risk, default risk, time to maturity, and other factors.

Uses

In a nutshell, time value calculations allow people to establish the future value of a given amount of money, at present. The present value (PV) is the money you have today. The future value (FV) is the accumulated amount of money you get after investing the original sum at a certain interest rate and for a given time period, say, 2 years. The concept has a wide range of applications in corporate financial matters-bonds, shares, loan facilities among others.

Fundamental formulas in time value of money calculations

Let,

FV = future value

PV = present value

r = interest rate

n = number of investment periods per year

t = number of years

Note: Besides annual interest payments, interest could be compounded monthly, quarterly or semi-annually. If for instance, interest is payable quarterly, then we have 12/3 i.e 4 investment periods per year.

$$PV = FV\left(1 + \frac{r}{n}\right)^{-n*t}$$

$$FV = PV\left(1 + \frac{r}{n}\right)^{n*t}$$

Question

Suppose an individual invests $10,000 in a bank account that pays interest at a rate of 10% compounded annually. What will be the future value after 2 years?

A. $12,000

B. $12,100

C. $22,000

Solution

The correct answer is B.

PV=10,000, r=0.1, n=1, t=2

$$\begin{aligned} FV &= 10,000(1+\frac{0.1}{1})^{1*2} \\ &= 10,000(1.1)^2 \\ &= 12,100 \end{aligned}$$

To confirm our answer, we could work out the PV of a future value of 12,100 invested under similar terms, starting with the FV of $12100.

$$\begin{aligned} PV &= 12,100(1+0.1/1)^{-(1*2)} \\ &= 12,100(1.1)^-2 \\ &= \$10,000 \end{aligned}$$

Points to note

First, establish all the components of the relevant formula before commencing actual calculation. Secondly, only the term within the brackets is subject to the power function.

The concept of time value of money serves as the foundation for more concrete financial calculations such as simple interest, compound interest, and the value of stocks/bonds at any given point in time.

Reading 6 LOS 6a:

Interpret interest rates as required rates of return, discount rates, or opportunity costs

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LOS 6b: explain an interest rate as the sum of a real risk-free rate and premiums that compensate investors for bearing distinct types of risk

Interest is basically a reward paid by a borrower, for the use of an asset, usually capital, belonging to a lender. It's the compensation paid for the loss of use of the asset. We could also describe it as the opportunity cost of alternative investments.

At the time of lending, the lender is most likely to have a portfolio of investment vehicles to choose from. As such, they must charge a premium for the 'loss' of the alternative investment opportunities. We express interest as an annual percentage, from which we can calculate monthly, quarterly or semi-annual equivalents. The level of interest rate is a function of several risks.

Interest Rate Formula

$$\begin{aligned} r = &\text{ risk free rate} + \text{inflation premium} \\ &+ \text{ liquidity premium} + \text{maturity premium} \\ &+ \text{default premium} \end{aligned}$$

Types of risks

The risk-free rate is the rate of return offered by assets largely considered risk free, usually government securities/local authority bonds. The reason is that governments can always come up with ways of repaying its debts, for instance, by increasing the tax payable by its citizens.

Inflation Risk

Inflation risk is the loss of purchasing power of money as a result of the increase in prices of consumer goods. A premium is added to the risk-free rate of interest to cushion investors from the loss of purchasing power of money. The aim is to offset the additional cash that investors will have to pay when buying goods.

Financial analysts measure inflation on a monthly or yearly basis. They use corresponding periods to calculate inflation. For instance, to establish the inflation in the month of June 2017, we have to compare the prices of selected consumer goods in June 2016 and June 2017.

Liquidity Risk

Liquidity risk refers to a situation where investors could find themselves unable to meet short-term liabilities as a result of investing their money in fixed term asset. You could find a very profitable business undergoing liquidation or foreclosure, simply because of insufficient liquidity.

Major investments usually have fixed durations and an investor cannot access funds until maturity. A financial analyst should set aside funds enough to cater for the day-to-day running of a company.

Default Risk

Default risk describes a situation where a borrower may cease to repay borrowed funds as a result of bankruptcy. This might result in significant losses on the side of the lender.

Maturity Risk

Lastly, maturity risk refers to the uncertainty associated with long-term investments. To protect themselves from these risks, lenders add a premium to the risk-free rate with respect to each risk.

Question

Which of the following statements is *most accurate* about interest rates?

A. The risk-free rate is the rate of return offered by assets largely considered risk free such as corporate bonds.

B. Inflation risk describes a situation where a borrower may cease to repay borrowed funds as a result of bankruptcy.

C. The interest rate formula is: Interest rate = risk-free rate + default premium + liquidity premium + inflation premium + maturity premium

Solution

The correct answer is C.

You must add the four types of risks to the risk-free rate to come up with the overall rate of interest, r.

Option A is incorrect. The risk-free rate is the rate of return offered by assets largely considered risk free such as government securities.

Option B is incorrect. Inflation risk is the loss of purchasing power of money as a result of the increase in prices of consumer goods.

Reading 6 LOS 6b

Explain an interest rate as the sum of real risk-free rate, and premiums that compensate investors for bearing distinct types of risk.

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LOS 6c: calculate and interpret the effective annual rate, given the stated annual interest rate and the frequency of compounding

The effective annual rate of interest (EAR) refers to the rate of return earned by an investor in a year, taking into account the effects of compounding. Remember, compounding is the process by which invested funds grow exponentially as a result of both the principal and the already accumulated interest, earning more interest. In other words, interest earned itself earns more interest. Mathematically, we may define EAR as follows:

$$\text{EAR} = (1 + \text{periodic rate})^{m} - 1$$

Where, periodic rate = $\frac{\text{stated annual rate}}{m}$

And *m* is the number of compounding periods per year.

Example 1

Suppose you're asked to calculate the EAR, given a stated annual rate of 10% compounded semi-annually. You would be expected to directly apply the above formula.

$$\text{EAR} = (1 + \text{periodic rate})^{m} - 1$$

Establishing the components already known,

Stated annual rate = 0.1;

m = 2

Periodic rate = 0.1/2 = 0.05

Hence,

$$\begin{aligned} \text{EAR} &= (1 + 0.05)^{2} - 1 \\ &= 10.25\% \end{aligned}$$

Example 2: A range of Compounding Frequencies

Using a stated annual rate of 12%, compute the Effective rates for daily, monthly, quarterly and semi-annual compounding periods.

$$\text{Semi-annual compounding} = (1 + 0.06)^2 - 1 = 0.1236 = 12.36\%$$
$$\text{Quarterly compounding} = (1 + 0.03)^4 - 1 = 0.12551 = 12.55\%$$
$$\text{Monthly compounding} = (1 + 0.01)^{12} - 1 = 0.12683 = 12.68\%$$
$$\text{Daily compounding} = (1 + 0.00032877)^{365} - 1 = 12.75\%$$

First, you should note that as the compounding frequency increases, so does the EAR.

Furthermore, the stated rate is equal to the EAR only when the interest is compounded annually.

Why is the effective annual rate of interest so important?

The EAR is an important concept in financial management as it is used to compare two or more projects that calculate compound interest differently. Assume that you have two projects X and Y. Project X pays 5% interest compounded monthly, while project Y pays 5% interest compounded quarterly. By calculating the EAR represented by each of these two rates, you would be able to pick the most profitable project of the two. Furthermore, the higher the EAR, the higher the return offered by an investment.

Question

John Ross, a financial analyst, would like to have $20,000 saved in his bank account at the end of 5 years. The bank offers a return of 10% per annum compounded semi-annually.

How much should Ross invest at the beginning so as to attain his goal?

A. $0.61

B. $12,279

C. $10,000

Solution

The correct answer is B.

The question asks us to find the present value of an accumulation of $20,000 that has been earned over a five-year period. As you will recall, the PV formula is:

$$PV = FV(1+r)^{-n}$$

Where FV is the future value,

r is the rate of return and,

n is the term of the investment

First, we have to determine the value of r, which will be the EAR

$$\begin{aligned} EAR &= \left\{1+\left(\frac{0.1}{2}\right)\right\}^{2} - 1 \\ &= (1+0.05)^{2} - 1 = 10.25\% \end{aligned}$$

Lastly, we calculate the PV.

$$PV = 20{,}000(1.1025)^{-5} = \$12{,}279$$

Reading 6 LOS 6c

Calculate and interpret the effective annual rate, given the stated annual interest rate and the frequency of compounding.

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LOS 6d: solve time value of money problems for different frequencies of compounding

Some types of investments are known to accumulate interest more than once a year. This results from semi-annual, quarterly, monthly or daily compounding. This, in turn, leads to different present values (PV) or future values (FV) of an investment depending on the frequency of compounding employed.

We have previously seen that the effective annual rate of interest increases as the number of compounding periods per year increases. In calculating the present value or future value of an investment with multiple compounding periods per year, the most important thing is to ensure that the interest rate used corresponds to the number of compounding periods present per year.

Future value

$$FV = PV\{(1+\frac{r_q}{m})\}^{m*n}$$

Where r_q is the quoted annual rate,

m represents the number of compounding periods (per year)

Lastly, n is the number of years

Present value

Suppose you make PV the subject of the above formula. You should find that

$$PV = FV\{(1+\frac{r_q}{m})\}^{-m*n}$$

Example: Present value with monthly compounding

Suppose you wish to have $10,000 in your savings account at the end of the next 3 years. Assume that the account offers a return of 9 percent per year subject to monthly compounding. How much would you need to invest now so as to have the specified amount after the three years?

Solution

First, we write down the formula to use,

$$PV = FV\{(1+\frac{r_q}{m})\}^{-m*n}$$

Secondly, we establish the components that we already have:

$r_q = 0.09$,   m = 12 since compounding is monthly, n = 3 years

Then, we factor everything into the equation to find our PV

$$\begin{aligned} PV &= 10,000\{(1+\frac{0.09}{12})\}^{-12*3} \\ &= 10,000 * 1.0075^{-36} \\ &= \$7,641.50 \end{aligned}$$

Therefore, you will need to invest at least $7,642 in your account to ensure that you have $10,000 after three years.

Question 1

Elizabeth Mary invests $2,000 dollars in a project that pays a rate of return of 8% compounded quarterly. How much interest will Mary have earned after investing in the project for two years?

A. $2,300

B. $2,343.32

C. $343.32

Solution

The correct answer is C.

$$\begin{aligned} FV &= 2000\{(1+\frac{0.08}{4})\}^{4*2} \\ &= 2,000 * 1.02^{8} \\ &= \$2,343.32 \end{aligned}$$

Therefore, interest gained = 2,343.32-2,000= $343.32

Question 2

What if the project paid a rate of return of 8% compounded daily? How much interest would Elizabeth Mary earn?

A. $2,347

B. $347

C. $2,340

Solution

The correct answer is B.

$$\begin{aligned} FV &= 2,000\{(1+\frac{0.08}{365})\}^{365*2} \\ &= 2,000 * 1.00021918^{730} \\ &= \$2,347 \end{aligned}$$

Similarly, the interest= 2,347-2,000= $347

You should notice that with a higher compounding frequency, the corresponding profit is also higher. This confirms that interest earned increases as the number of compounding periods per year increases.

Note

We could have converted our stated annual rates into the effective annual rate of interest, and arrive at similar answers. However, if you do that, you should ensure that you use years in the computation.

Reading 6 LOS 6d

Solve time value of money problems for different frequencies of compounding.

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